Comp 254, Image Processing and Analysis
Spring 2002, Prof. Guido Gerig

Solutions to Assignment 3: Hough Transform

Required Readings: Generalizing the Hough Transform ..., D.H. Ballard 1981
Use of Hough Transformation to detect Lines ..., Duda and Hart
Linking Image Space and Accumulator-Space, G. Gerig, ICCV'87

I. Theoretical Problems

Problem 1: HT as a point to curve transformation

Show that points that lie on the same curve $\rho = x\cos\theta + y\sin\theta$ in parameter space with axes (ρ, θ) all represent straight lines in image space which intersect at a specific image point (x_0, y_0).

You can solve this problem either by showing that two straight lines intersect at the point (x_0, y_0) or by demonstrating that an arbitrary straight line passes the point (x_0, y_0). We choose the latter:
A point (x_0, y_0) in image space transforms into a curve in parameter space [1] :

$$(x_0, y_0) \rightarrow \boldsymbol{\rho} = x_0 \cos\boldsymbol{\theta} + y_0 \sin\boldsymbol{\theta} \tag{1}$$

A point (θ_0, ρ_0) that solves the equation (1) represents a straight line in image space

$$\rho_0 = \mathbf{x}\cos\theta_0 + \mathbf{y}\sin\theta_0 \tag{2}$$

Putting the original point (x_0, y_0) into equation 2 we get $\rho_0 = x_0\cos\theta_0 + y_0\sin\theta_0$. This equation holds since (θ_0, ρ_0) solves equation (1). Therefore, the straight line passes the point (x_0, y_0), and since the parameters (θ_0, ρ_0) were arbitrary, this holds for all the straight lines that intersect at the point (x_0, y_0).
An elegant solution has been presented by Brian Salomon (CS, Spring 2002):
Using the equation $y = \frac{\rho - x\cos\theta}{\sin\theta}$, and knowing that ρ is dependent upon θ, we have:

$$y = \frac{x_0\cos\theta + y_0\sin\theta - x\cos\theta}{\sin\theta} = x_0\cot\theta + y_0 - x\cot\theta.$$

It is clear that if $x = x_0$ then for any value of $theta:\ y = y_0$.

Problem 2: Compensation for errors

The quantization is chosen so that the straight line g_0 is given by the center of the parameter cell θ_0, ρ_0 (Fig. 1a). The error region in image space is bounded by the straight lines that are described by the corner points of the parameter cell $(\theta_0 \pm \frac{\Delta\theta}{2}, \rho \pm \frac{\Delta\rho}{2})$ (Fig. 1b).

[1] variables are denoted with **bold** letters

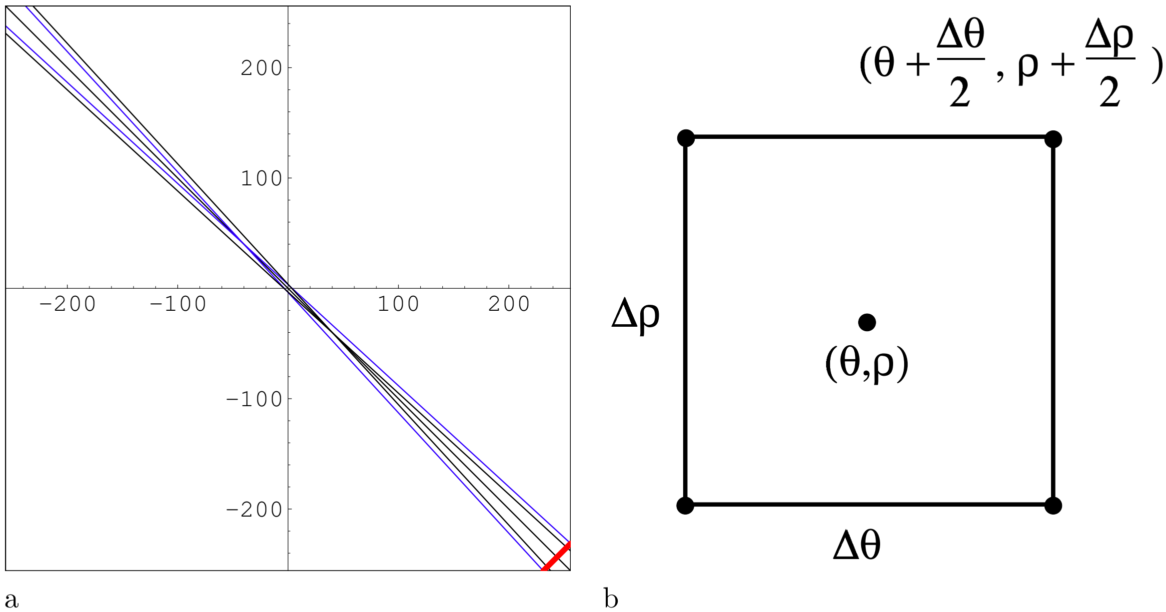

Figure 1: Error region in image space (a) defined by the quantized parameter cell (b).

Origin at center

We choose the center of the image as the origin of the coordinate system, so that $\theta_0 = 45^o$ and $\rho_0 = 0$.
The straight line given by $(-\frac{\Delta\rho}{2}, 45^o + \frac{\Delta\theta}{2})$ intersects the image border ($x = -256$) at the lowest vertical position, which is $(-256, 250.17)$ for the quantization $(1^o, 2$ Pixel) and $(-256, 231.19)$ for the quantization $(5^o, 5$ Pixel). The opposite points can be obtained by mirroring these points about the diagonal. The longest straight line pieces orthogonal to the diagonal can be calculated by Pythagoras and result in lengths of 8.247 and 35.087 pixels, respectively.

Origin at lower left

Putting the origin at lower left changes the geometry (see Figure 2). The horizontal intersect with a 512x512 image for the quantization $(5^o, 5$ pixels) become $(487.66, 0) or (512 - 24.34)$ and $512, 24.28)$. This indicates that these intersections are no longer symmetric (as in the case of the centered origin). Approximating the longest orthogonal line still as the length between intersection points, we get lengths of 8.247 and 35.093 pixels, respectively. Correct calculations would have to take the longest orthogonal line between these intersection points and the original line.
These values obtained with lower left centr are not very different from a centered version, but the shape of the error region is unsymmetric and can create very distorted clusters.

Problem 3: Detection of Ellipses

see paper Ballard and Brown (handout) that describes procedures with and without contour angle in details.

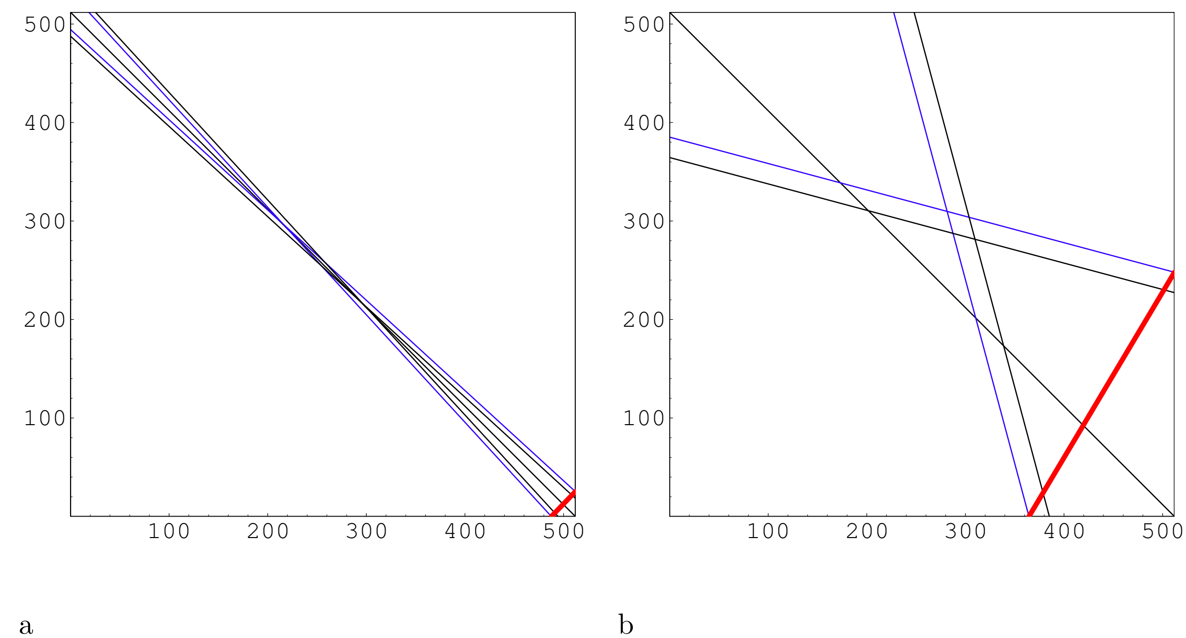

Figure 2: Error region in image space for the quantization ($5^o, 5$ pixels) and original lower left. The right Figure shows a quantization of ($10^o, 60$ pixels) to illustrate the asummetry of line intersections.