Weight and See

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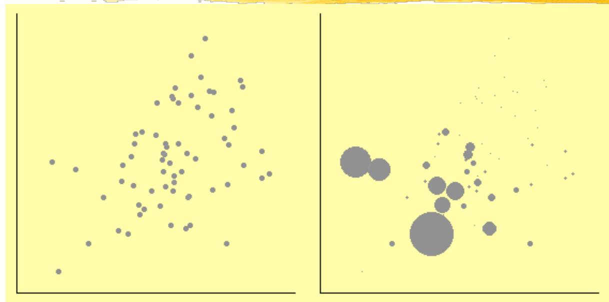
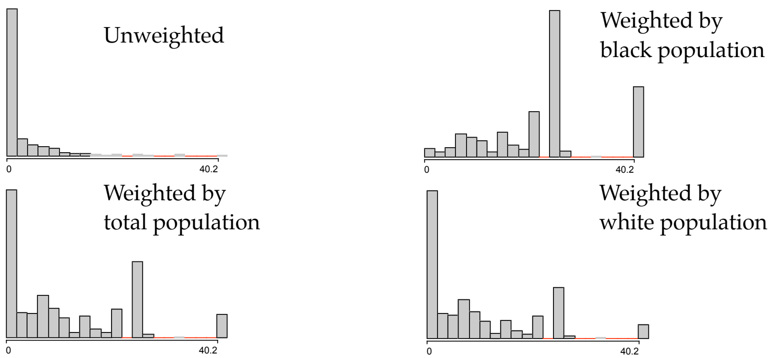
US Elections 2000 / 2004



% change in Bush vote between 2000 and 2004 v. Bush % in 2000 for the electoral districts in Florida
Unweighted on the left, weighted by votes cast 2004 on the right.

Weighting by population

Percentage black population in 437 MidWest counties



Weighting?

- Frequency weighting
- Probability weighting
- Importance weighting
- Analytic weighting
- (Other kinds of weighting)

Weighted graphics in R?

- Counts of "weight", "wtd"
  - Paul Murrell's book 0
  - Hadley Wickham's Ph D 1
- Google within R
  - "weighted histogram" 1
  - "weights" 2350
  - "weighted" 4860

An illustrative example

There are M groups with populations

$$N_j \quad \text{where} \quad j = 1, \ldots, M \quad \text{and} \quad \sum_{j=1}^{M} N_j = N.$$

Simple random samples are taken from each group of sizes

$$n_j \quad \text{where} \quad j = 1, \ldots, M \quad \text{and} \quad \sum_{j=1}^{M} n_j = n.$$

Each person i sampled from group j is checked to see if they are ill or healthy.

$$Z_{ij} = \begin{cases} 1 & \text{if ill,} \\ 0 & \text{if healthy,} \end{cases} \quad \text{with } i = 1, \ldots, n_j \text{ and } j = 1, \ldots, M$$

Frequency weighting (FW)

- Multiple independent inviduals have the same values
- Typically multivariate categorical data
- e.g., the Berkeley discrimination dataset either 4526 cases (Gender, Admit, Dept) or 24 combinations G(2) x A(2) x D(6)

FW example

Let m_{kj} denote the number of persons in group j with health k, i.e.

$$m_{kj} = \sum_{i=1}^{n_j} (Z_{ij} = k)$$

then the dataset can be written in the form

$$(k, j, m_{kj}) \quad \text{where } k \in \{0, 1\} \quad \text{and} \quad j = 1, \ldots, M.$$

The values m_{kj} are then frequency weights:

$$w_{kj}^{FW} = m_{kj} \quad \text{with} \quad m_{0j} + m_{1j} = n_j$$

Probability weighting (PW)

- When not all cases have the same probability of being sampled
  - stratified sampling
  - cluster sampling
  - adjusting for non-response
- Weighting is the inverse of the probability of being sampled

PW example

In the illustrative example, the dataset $\{Z_{ij}\}$ with $i = 1, \ldots, n_j$ and $j = 1, \ldots, M$ has n entries and probability weights that are inversely proportional to their sampling probability:

$$w_{ij}^{PW} \propto \frac{1}{n_j/N_j} = \frac{N_j}{n_j}$$

If individual i has probability p_i of appearing in the sample, then his weight

$$w_i^{PW} \propto 1/p_i$$

The weights may be normalised so that they sum to 1 or to an ‘equivalent sample size’ n.

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Importance weighting (IW)

- Cases have different importance
  - companies weighted by turnover or market capitalisation
  - market share changes weighted by shares
  - regional morbidity rates weighted by population
  - medical side-effect rates weighted by seriousness of effect

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IW example

In the illustrative example, the dataset of illness rates

$$\left\{ r_j = \frac{\sum_{i=1}^{n_j} Z_{ij}}{n_j} \right\}$$

has importance weights

$$w^{IW}(r_j) = N_j$$

In this case it is interesting to note that

$$\sum_{i=1}^{n_j} w_{ij}^{PW} \propto w^{IW}(r_j)$$

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Analytic weighting (AW)

- Combining results from studies
  - e.g., Metanalysis
- Weighting is the inverse of the variance (precision) or of the standard deviation
- cf. Bayesian modelling

AW example

Consider again the dataset of illness rates

$$\left\{ r_j = \frac{\sum_{i=1}^{n_j} Z_{ij}}{n_j} \quad \text{for } j = 1, \ldots, M \right\}.$$

Assuming an underlying binomial model $Z_{.j} \sim B_{n_j, p_j}$, gives analytic weights that are inversely proportional to variances:

$$w_j^{AW}(r_j) \propto \frac{n_j}{r_j(1 - r_j)}.$$

(If $N_j >> n_j$ there is no need to correct for finite sampling.)
Assuming $r_j \approx r$ for all j leads to

$$w_j^{AW} \propto n_j = w_j^{FW}$$

Properties of weights

- Additive
- Non-negative
- No missing values in the weighting variable
- Weights have to be interpretable

Types of graphic

- Point graphics
  - dotplots, glyphs, scatterplots, rotating plots, parallel coordinate plots, graphs
- Area graphics
  - histograms, barcharts, mosaicplots, missing value plots

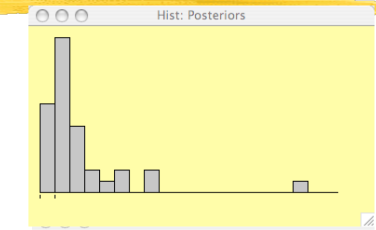
    (And what is a boxplot?)

Weighted area plots

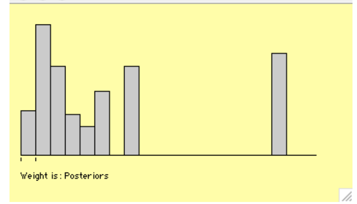
- Barcharts and spineplots
- Histograms and spinograms
- Mosaicplots and their variants
  - multiple barcharts
  - fluctuation diagrams
  - same binsize
  - doubledecker

Self-weighted histograms

Posterior probabilities of 36 models in a Bayesian Model Averaging analysis



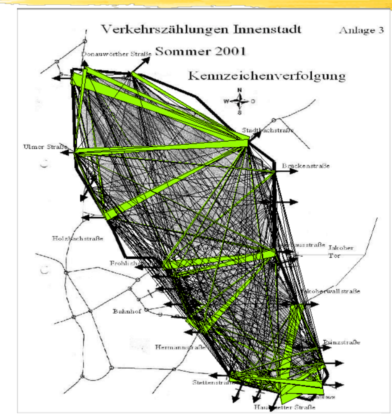
The posterior probabilities distribution weighted by the probabilities



Graphics for metanalysis

- Study results should be weighted by precision
- Displaying CIs for results gives more space to less precise studies (forest plots)
- Funnel plots would benefit from including precision with weight represented by size

Traffic flows



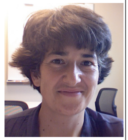
Augsburg city centre traffic flows superimposed on a map of the city.

Flows are represented by triangles whose base is proportional to the flow.

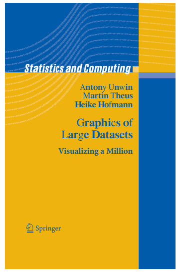
Software

- Mondrian
  - all platforms
  - stats.math.uni-augsburg.de/Mondrian/

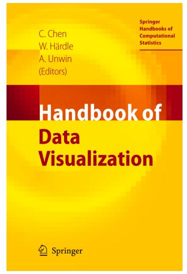
- MANET
  - (only for Macintosh)
  - stats.math.uni-augsburg.de/MANET/



More on graphics



and



Summary comments

- Weighted graphics should be considered more often
- There are different kinds of weighting with different interpretations
- Importance weighting offers most flexibility and requires most care
- Weighted plots provide additional insights